

Please write clearly in block capitals.	
Centre number	Candidate number
Surname	
Forename(s)	
Candidate signature	

# A-level **MATHEMATICS**

Unit Further Pure 2

Friday 22 June 2018

Morning

Time allowed: 1 hour 30 minutes

## **Materials**

For this paper you must have:

the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

### Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer all questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question.
  If you require extra space, use an AQA supplementary answer book; do not use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

## Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

#### Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Exam	iner's Use
Question	Mark
1	
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6	
7	
8	
9	
TOTAL	



# Answer all questions.

Answer each question in the space provided for that question.

**1 (a)** Given that  $f(r) = \frac{1}{(2r+3)(2r+5)}$ , show that

$$f(r-1) - f(r) = \frac{k}{(2r+1)(2r+3)(2r+5)}$$

where k is an integer.

[2 marks]

(b) Use the method of differences to find  $\sum_{r=1}^{N} \frac{1}{(2r+1)(2r+3)(2r+5)}$ .

[3 marks]

QUESTION PART REFERENCE	Answer space for question 1



QUESTION PART REFERENCE	Answer space for question 1
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- **2 (a)** The complex number  $-2\sqrt{2}+2\sqrt{6}\,\mathrm{i}$  can be expressed in the form  $r\mathrm{e}^{\mathrm{i}\theta}$ , where r>0 and  $-\pi<\theta\leqslant\pi$ .
  - (i) Show that  $r = (\sqrt{2})^n$  where n is an integer.

[2 marks]

(ii) Find the exact value of  $\theta$ .

[1 mark]

(b) Hence solve the equation  $z^5=-2\sqrt{2}+2\sqrt{6}\,\mathrm{i}\,$  giving your answers in the form  $r\mathrm{e}^{\mathrm{i}\theta}$ , where r>0 and  $-\pi<\theta\leqslant\pi$ .

[5 marks]

QUESTION PART REFERENCE	Answer space for question 2



QUESTION PART REFERENCE	Answer space for question 2
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**3** The sequence  $u_1$ ,  $u_2$ ,  $u_3$ , ... is defined by

$$u_1 = -1$$
,  $u_{n+1} = \frac{u_n - 5}{3u_n - 7}$ 

Prove by induction that  $u_n = \frac{2^{n+1}-5}{2^{n+1}-3}$ , for all integers  $n \geqslant 1$ .

[6 marks]

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QUESTION PART REFERENCE	Answer space for question 3
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4 (a)	Express $(1 + e^{2x})(1 + e^{-2x})$ in terms of $\cosh x$ .	
		[3 marks]

**(b)** Hence, find the value of

$$\int_0^1 \frac{1}{(1+e^{2x})(1+e^{-2x})} \, \mathrm{d}x$$

giving your answer in terms of e.

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QUESTION PART REFERENCE	Answer space for question 4



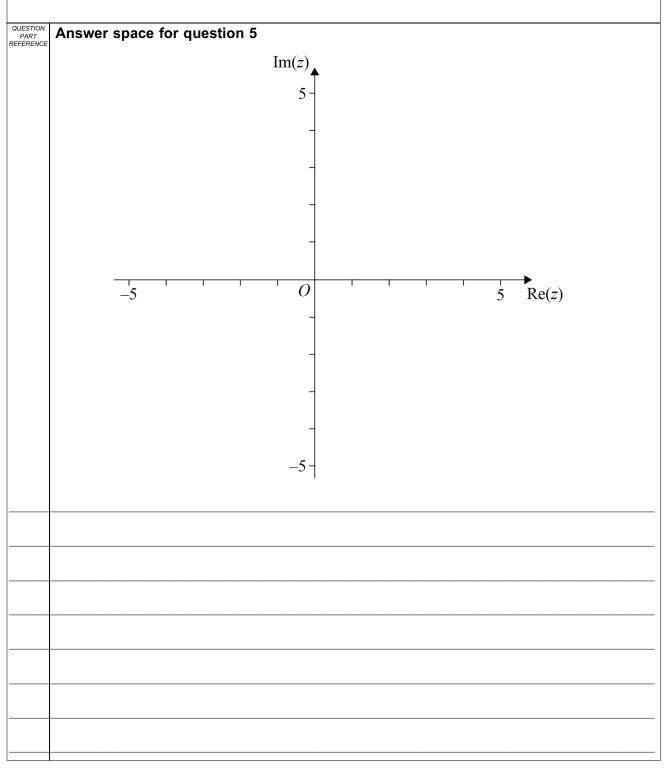
5 (a) The locus of points L, representing the complex number z, satisfies the equation

$$|z-2| = |z+4i|$$

 ${\bf Sketch}\; L \; {\bf on \; the \; Argand \; diagram \; below}.$ 

[3 marks]

(b) Given that  $|z-2|=|z+4\mathrm{i}|$ , find the complex number  $z_1$  for which |z| has its least value.





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**6 (a)** Sketch the graph of  $y = \cosh^{-1} x$  on the axes below.

[2 marks]

**(b)** Given that  $y = \cosh^{-1} x$ , show that  $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\sqrt{x^2 - 1}}$ .

[3 marks]

(c) A curve has equation  $y = \frac{5}{3} - 4x + \cosh^{-1}(3x)$ . Show that the curve has a single stationary point, M, and express the y-coordinate of M as a natural logarithm.

[5 marks]

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7 A curve is defined parametrically by

$$x = 3 - \cos 2t, \qquad y = 4\sin t$$

The arc of the curve from t=0 to  $t=\frac{\pi}{2}$  is rotated through  $2\pi$  radians about the x-axis to generate a surface with area S.

(a) Show that  $S = k\pi \int_0^{\frac{\pi}{2}} \sin t \cos t \sqrt{(1+\sin^2 t)} \, dt$ , where k is an integer.

[5 marks]

(b) Hence find the value of S, giving your answer in the form  $\frac{\pi}{3}(m\sqrt{2}+n)$ , where m and n are integers.

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QUESTION PART REFERENCE	Answer space for question 7



QUESTION PART REFERENCE	Answer space for question 7
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8 The cubic equation  $2z^3 + 5z + 3 = 0$  has roots  $\alpha$ ,  $\beta$  and  $\gamma$ .

- (a) Write down the value of:
  - (i)  $\alpha\beta + \beta\gamma + \gamma\alpha$ ;

[1 mark]

(ii)  $\alpha\beta\gamma$ .

[1 mark]

**(b)** Find the value of  $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ .

[2 marks]

(c) (i) Use the substitution  $z^2 = \frac{1}{x}$  to show that

$$9x^3 - 25x^2 + mx + n = 0$$

where m and n are integers.

[4 marks]

(ii) Hence find the value of  $\frac{1}{\alpha^4} + \frac{1}{\beta^4} + \frac{1}{\gamma^4}$ .

PART REFERENCE	Answer space for question 8



QUESTION PART REFERENCE	Answer space for question 8
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9 (a) Use de Moivre's theorem to show that

$$\cos 5\theta = 16\cos^5\theta + A\cos^3\theta + B\cos\theta$$

where A and B are integers.

[5 marks]

(b) (i) Given that  $\cos 5\theta=0$  and  $\cos \theta \neq 0$ , find the possible values of  $\cos^2 \theta$ , giving your answers in simplified surd form.

[2 marks]

(ii) Hence find the exact value of  $\cos^2\frac{3\pi}{10}$  and deduce that  $\cos\frac{3\pi}{5}=\frac{1-\sqrt{5}}{4}$ .

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## **END OF QUESTIONS**

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